UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2008 question paper

9709 MATHEMATICS

9709/01 Paper 1, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2008 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

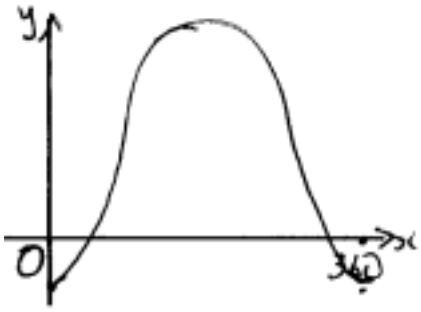
SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

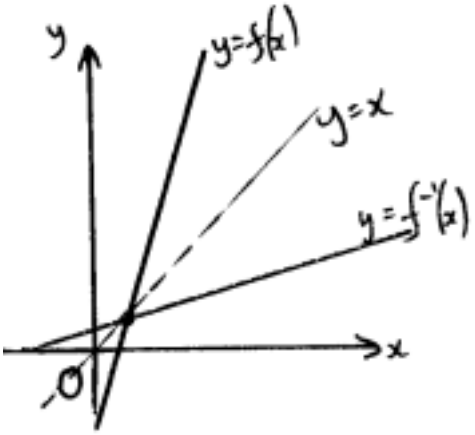
MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1 $\left(\frac{x}{2}+\frac{2}{x}\right)^6$ Term in $x^2\left(\frac{x}{2}\right)^4\left(\frac{2}{x}\right)^2\times 15$ Coeff $=\frac{15}{4}$ or 3.75	M1 A1 A1 [3]	Correct term – needs powers 4 and 2 For × 15 Ignore inclusion of x^2
2 $\frac{1+\sin x}{\cos x}+\frac{\cos x}{1+\sin x}\equiv\frac{2}{\cos x}$ LHS $\frac{(1+\sin x)^2+\cos^2 x}{\cos x(1+\sin x)}$ $=\frac{2+2\sin x}{\cos x(1+\sin x)}$ $=\frac{2}{\cos x}$	M1 M1 A1 A1 [4]	Reasonable algebra. Correct denominator and one term correct in numerator Use of $\sin^2 x+\cos^2 x=1$ For $2+2\sin x$ Co – answer was given – check preceding line
3 1st term $= a = 6$ 5th term $= a + 4d = 12$ $\rightarrow d = 1.5$ $S_n = \frac{n}{2}(12+(n-1)1.5) = 90$ $\rightarrow n^2+7n-120=0$ $\rightarrow n = 8$	B1 M1 DM1 A1 [4]	Correct value of d Use of correct formula with his d Correct method for soln of quadratic Co (ignore inclusion of $n = -15$)
4 (i) $\overrightarrow{PA} = -6\mathbf{i}-8\mathbf{j}-6\mathbf{k}$ $\overrightarrow{PN} = 6\mathbf{i}+2\mathbf{j}-6\mathbf{k}$	B1 B2, 1 [3]	Co – column vectors ok One off for each error (all incorrect sign – just one error)
(ii) $\overrightarrow{PA}.\overrightarrow{PN} = -36-16+36=-16$ $\cos APN = \frac{-16}{\sqrt{136}\sqrt{76}}$ $\rightarrow APN = 99°$	M1 M1 M1 A1 [4]	Use of $x_1x_2+y_1y_2+z_1z_2$ Modulus worked correctly for either one Division of "–16" by "product of moduli" Allow more accuracy

5 $x \mapsto a - b\cos x$ **(i)** $a + b = 10$ and $a - b = -2$ $\rightarrow a = 4$ and $b = 6$	M1 A1 Al [3]	M1 for either correct. A1 both correct Co (if $a - b = 10$, and $a + b = -2$, treat as MR –1, **(i)** $a = 4$, $b = -6$, **(ii)** 131.8, 228.2, **(iii)** Sketch is mirror image in $y = 4$)
(ii) $4 - 6\cos x = 0$ $\rightarrow \cos x = 2/3$ $\rightarrow x = 48.2°$ or $311.8°$	M1 A1 A1√ [3]	Makes cos$x$ subject and uses inv cos. For 1st angle. √ for 360° – "his angle"
(iii) 	B2,1 [2]	Must be just one cycle Starts at – 2 and ends at –2 Max at 10. "V shapes " lose a mark. Parabolas lose 1 mark.
6 **(i)** Using $s = r\theta$, $9 = 5\theta \rightarrow \theta = 1.8$ rad.	Ml Al [2]	Use of formula. co
(ii) Uses *POT*. Halves the angle Uses tangent in *POT* $PT = 5\tan 0.9 = 6.30$ cm (not 6.31)	Ml Ml Al [3]	Realises the need to halve Use of tangent – even if angle not halved co
(iii) area of sector = ½ × 5^2 × 1.8 (22.5) Area of *POT* = ½ × 5 × 6.30 (15.75) Shaded area = 2 triangles – sector $\rightarrow$ 9.00 (allow 8.95 to 9.05)	Ml Ml Al [3]	Use of $A = \frac{1}{2}r^2\theta$ with 1.8 or 0.9. Use of ½bh and (2 triangles – sector) co
7 **(i)** $4x + 2\pi r = 80$ $A = x^2 + \pi r^2$ $\rightarrow A = \dfrac{(\pi + 4)x^2 - 160x + 1600}{\pi}$	B1 B1 M1 A1 [4]	Connection of lengths Connection of areas Eliminates r. co but answer given.
(ii) $\dfrac{dA}{dx} = \dfrac{2(\pi + 4)x - 160}{\pi}$ $= 0$ when $x = \dfrac{160}{2(\pi + 4)}$ or 11.2	Ml Al DM1 Al [4]	Attempt at differentiation. co Ignore omission of π. Sets to 0 and solves. co

8 $y = 5 - \frac{8}{x}$, $P(2, 1)$		
(i) $\frac{dy}{dx} = \frac{8}{x^2}$	B1	Correct differentiation
m of tan = 2 m of normal = –½	M1	Use of $m_1 m_2 = -1$
Eqn of normal $y - 1 = -\frac{1}{2}(x - 2)$	M1	Correct method for line
$\rightarrow 2y + x = 4$	A1 [4]	Answer given
(ii) Sim eqns $2y + x = 4$, $y = 5 - \frac{8}{x}$		
$\rightarrow x^2 + 6x - 16 = 0$ or $y^2 - 7y + 6 = 0$	M1	Complete elimination of x or y
$\rightarrow (-8, 6)$	DM1 A1 [3]	Soln of quadratic. co
(iii) Length = $\sqrt{10^2 + 5^2} = \sqrt{125}$	M1	Correct use of Pythagoras
$\rightarrow$ 11.2 (accept $\sqrt{125}$ or $5\sqrt{5}$ etc)	Al [2]	For his points.
9 $y = \sqrt{3x + 1}$		
(i) A = $\int x \, dy = \int_1^2 \frac{y^2 - 1}{3} dy$	M1	Uses integration wrt y
$= \left[\frac{y^3}{9} - \frac{y}{3}\right] = \frac{4}{9}$ (allow 0.44 to 0.45)	Al DM1 A1 [4]	Integration correct Use of limits 0 to 1. co
[or $2 - \int \sqrt{3x+1} \, dx = [2 - \frac{(3x+1)^{\frac{3}{2}}}{\frac{3}{2} \times 3}] = \frac{4}{9}$]		B1 for everything but ÷3. B1 for ÷3. M1 for "2–" and use of limits 0 to 1.
B1 B1 M1A1		
(ii) $V = \pi \int y^2 dx = \pi \int (3x + 1) dx$	M1	M1 for correct formula used with y^2 and integration wrt x. (does not need π)
$= \pi\left(\frac{3x^2}{2} + x\right)$ from 0 to 1	A1	A1 integration correct, including π.
Vol of cylinder = $\pi \times 2^2 \times 1 = 4\pi$	B1	Or by integration of $y^2 = 4$
$\rightarrow$ Subtraction $\rightarrow$ 1.5 π (4.71)	A1 [4]	co

(iii) $\frac{dy}{dx} = \frac{1}{2}(3x+1)^{-\frac{1}{2}} \times 3$	B1 M1	B1 for everything but × 3. M1 for × 3.
If $x = 0$, $m = \frac{3}{2}$. If $x = 1$, $m = \frac{3}{4}$		
At $x = 0$, angle = 56.3°	M1	Linking angle with tangent once
At $x = 1$, angle = 36.9°		
→ angle between = 19.4°	A1 [4]	co
Could use vectors, or tan($A - B$) formula. Could also find tangents, point of intersection, 3 lengths and cosine rule.		
10 $f : x \mapsto 3x - 2$		
(i) 	B1	Graph of $y = 3x - 2$
	B1 [2]	Evidence of mirror image in $y = x$ or graph of 1/3 (x + 2). Whichever way, there must be symmetry shown or quoted or implied by same intercepts.
(ii) $gf(x) = 6(3x - 2) - (3x - 2)^2$	M1	Must be gf, not fg
$= -9x^2 + 30x - 16$	A1	Co
$d/dx = -18x + 30$	M1	Differentiates or completes square
= 0 when $x = 5/3$	DM1	Sets to 0, solves and attempts to find y
→ Max of 9	A1 [5]	All ok – answer was given
($gf(x) = 9 - (3x - 5)^2$ → Max 9)		
(iii) $6x - x^2 = 9 - (x - 3)^2$	B1, B1 [2]	Does not need a or b.
(iv) $y = 9 - (3 - x)^2$	M1	Order of operations in making x subject
$3 - x = \pm\sqrt{9 - y}$	DM1	Interchanging x and y
→ $h^{-1}(x) = 3 + \sqrt{(9 - x)}$	A1 [3]	Allow if ± given (Special case → if correct with y instead of x, give 2 out of 3)